Int J Theor Phys (2008) 47: 2716–2721
DOI 10.1007/s10773-008-9710-5

Photon-Added Entangled Coherent State

Q. Dai · H. Jing

Received: 15 January 2008 / Accepted: 12 March 2008 / Published online: 25 March 2008
© Springer Science+Business Media, LLC 2008

Abstract We propose a feasible scheme to create the single-mode or two-mode photon-added entangled coherent state (PAECS) and then compare its entanglement degree with that of the entangled coherent state or even the photon-added-and-subtracted entangled coherent state.

Keywords Entangled coherent state · Single-photon state · Quantum entanglement

1 Introduction

The preparation and manipulations of the nonclassical or entangled quantum states play an essential role in current quantum information science [1–10]. Many novel schemes have been proposed in the past decade to create various conceptually new quantum state such as the entangled coherent state (ECS) [7] and the photon-added coherent state (PACS) [8]. By using a type-I beta-barium borate (BBO) crystal, a single-photon detector (SPD) and a balanced homodyne detector, Zavatta et al. in their recent beautiful experiment produced a single-photon-added coherent state (SPACS) and thereby first visualized the classical-to-quantum transition process [9]. Very recently, the basic quantum (bosonic) commutation relation was elegantly demonstrated by optically probing the different properties of the SPACS, the photon-added-and-subtracted coherent state, and the bare coherent state [10].

In this paper, we propose a feasible scheme to create the photon-added entangled coherent state (PAECS) and then demonstrate the different entanglement degrees of the PAECS, the bare ECS, and even the photon-added-and-subtracted ECS. The success probability of the SPACS generation in our scheme is also shown to be largely enhanced due to the conditional preparation of a discrete-variable entangled W state [11–13] or a hybrid-variable entangled single-photon-added coherent state.

Q. Dai
Department of Physics, Xinyang Normal University, Xinyang 464000, People's Republic of China

H. Jing (✉)
Department of Physics, Henan Normal University, Xinxiang 453007, People's Republic of China
e-mail: jinghui73@gmail.com

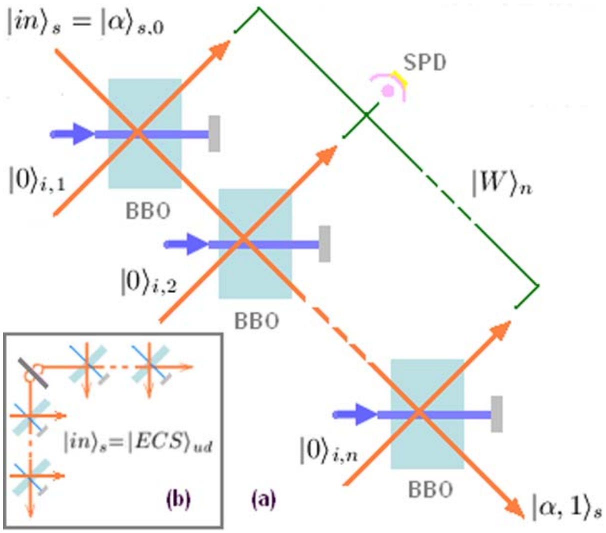

Fig. 1 (Color online) Schematic diagram for the enhancement of SPACS generation by conditionally preparing the discrete-variable (DV) entangled W state in two different channels. The input signal is in (**a**) a classical coherent state or (**b**) an entangled coherent state (ECS) [7]

2 Photon-Added Coherent State

The PACS $|\alpha, m\rangle$, firstly introduced by Agarwal and Tara [8], is defined as the successive one-photon excitations on a classical coherent-state light: $|\alpha, m\rangle = N_m \hat{a}^{\dagger m}|\alpha\rangle$, where $\hat{a}$ ($\hat{a}^\dagger$) is the photon annihilation (creation) operator and N_m is a normalized constant [8]. Obviously, for $\alpha \to 0$ or $m \to 0$, $|\alpha, m\rangle$ reduces to the purely quantum Fock state or the classical coherent state. In the experiment of Zavatta et al. [9], the SPACS was produced via the parametric down-conversion process of quantum optics in which one pump photon is converted into one signal and one idler photon. For a classical pumping, this process can be briefly described by transforming an input coherent signal $|\psi(0)\rangle_1 = |\alpha\rangle_s|0\rangle_i$ into an output state, after an effective interaction time λ,

$$|\psi(t)\rangle_s = \exp[\lambda(\hat{a}_s^\dagger \hat{a}_i^\dagger - \hat{a}_s\hat{a}_i)]|\psi(0)\rangle_1 \approx |\alpha\rangle_s|0\rangle_i + \lambda|\alpha, 1\rangle_s|1\rangle_i, \tag{1}$$

in short-time limits $\lambda \ll 1$. Clearly, when a single photon is detected in the output idler, a SPACS is created in the output signal with a success probability being proportional to $p_1^1 = |\lambda|^2(1 + |\alpha|^2)$. Along this line, one can also create the general PACS or m-photon-added coherent state.

As shown in Fig. 1(a), a series of m identical optical parametric amplifiers can be combined to reach that goal. Without losing generality, we simply assume the same low-gain and the same classical pumps. Therefore, an input coherent signal (with vacuum idlers) is converted to

$$|\psi(t)\rangle_m = \prod_{j=1}^{m} \exp[\lambda(\hat{a}_{s,j}^\dagger \hat{a}_{i,j}^\dagger - \hat{a}_{s,j}\hat{a}_{i,j})]|\psi(0)\rangle_m, \tag{2}$$

from which the PACS $|\alpha, m\rangle$ can be obtained with the success probability $p_m^j = N|\lambda^j|^2 j! \times L_j(-|\alpha|^2)$, where L_j is the Laguerre polynomial (j is an integer) [8]. Clearly for $j = 1$, we have $p_m^1 = mp_1^1$. This means that, comparing with the single-BBO scheme, the success probability of the SPACS creation via this m-BBO scheme can be greatly enhanced, at least in principle.

This improvement of the SPACS creation is actually due to the conditional formation of the N-qubit W entangled state in the idlers. Concretely, by achieving a SPACS signal, one has in the idlers

$$|\psi(t)\rangle_{\text{idler}} = \frac{1}{\sqrt{m}}(|1\underbrace{00\ldots0}_{m-1}\rangle + |01\underbrace{00\ldots0}_{m-2}\rangle + \cdots + |\underbrace{00\ldots0}_{m-1}1\rangle). \quad (3)$$

This means that the SPACS can be achieved if any one of the m detectors captures one photon. In other words, the SPACS signal itself can *not* distinguish which one of the detectors got a photon. We note that, comparing with the single-BBO scheme, the improvement effect here also exist for creating a general PACS or m-photon-added coherent state. This effect is useful in view of the extreme difficulty to achieve a large nonlinear susceptibility in the ordinary nonlinear optics mediums [1, 14–18].

3 Photon-Added Entangled Coherent State

As is shown in Fig. 1(b), the above repeated BBO method can be directly applied to some more complicated scheme such as an input ECS signal [6, 7]. For a concrete example, we consider two BBO in the upper-channel and one BBO in the down-channel. The initial state with a two-mode ECS signal can be written as $|\psi\rangle_{\text{in}} = |ECS\rangle_{ud}|0\rangle_{ui,1}|0\rangle_{ui,2}|0\rangle_{di}$, where the Sanders ECS [7] is: $|ECS\rangle_{ud} = \frac{1}{\sqrt{2}}[e^{-i\pi/4}|i\beta\rangle_{us}|i\alpha\rangle_{ds} + e^{i\pi/4}|-\alpha\rangle_{us}|\beta\rangle_{ds}]$. Then, if only one (upper or lower) detector clicks, we can get the output state (up to first order of λ)

$$\begin{aligned} |\psi\rangle_{\text{out}} &= |\psi\rangle_{\text{in}} + \lambda|SMECS\rangle^{I}_{ud}|EPR\rangle_{u,i}|0\rangle_{di} \qquad \text{(upper-click)}, \\ |\psi\rangle_{\text{out}} &= |\psi\rangle_{\text{in}} + \lambda|SMECS\rangle^{II}_{ud}|0\rangle_{ui,1}|0\rangle_{ui,2}|1\rangle_{di} \quad \text{(lower-click)}, \end{aligned} \quad (4)$$

where $|EPR\rangle$ and $|SMECS\rangle$ respectively denote the EPR state and the single-mode-excited ECS

$$\begin{aligned} |SMECS\rangle^{I}_{ud} &= \frac{1}{\sqrt{2}}\big[e^{-i\pi/4}|i\beta,1\rangle_{us}|i\alpha\rangle_{ds} + e^{i\pi/4}|-\alpha,1\rangle_{us}|\beta\rangle_{ds}\big], \\ |SMECS\rangle^{II}_{ud} &= \frac{1}{\sqrt{2}}\big[e^{-i\pi/4}|i\beta\rangle_{us}|i\alpha,1\rangle_{ds} + e^{i\pi/4}|-\alpha\rangle_{us}|\beta,1\rangle_{ds}\big], \end{aligned} \quad (5)$$

and $|EPR\rangle_{u,i} = |1\rangle_{ui,1}|0\rangle_{ui,2} + |0\rangle_{ui,1}|1\rangle_{ui,2}$. Therefore, no matter which detector captures one photon, the single-mode ECS always can be obtained. The only difference is the creation of the two-mode EPR entangled state or the single-photon state in the idlers for the upper-click or the lower-click case, leading to the SMECS success probability in the former case being 2 times than in the later case.

The SMECS in (5) is a single-mode single-photon-added ECS. Similarly, if one have two clicks from two upper detectors, the single-mode two-photon-added ECS can also be obtained. More interestingly, if one have the two clicks from one upper detector and one lower detector, another kind of two-mode two-photon-added ECS can be produced, i.e.,

$$|TMECS\rangle_{ud} = \frac{1}{\sqrt{2}}\big[e^{-i\pi/4}|i\beta,1\rangle_{us}|i\alpha,1\rangle_{ds} + e^{i\pi/4}|-\alpha,1\rangle_{us}|\beta,1\rangle_{ds}\big], \quad (6)$$

which for $\beta = 0$ and large α can be written as a hybrid-variable entangled state: $|\Psi^{\rho}_{a,b}\rangle = |1\rangle_a|\alpha\rangle_b + e^{i\rho}|\alpha\rangle_a|\tilde{1}\rangle_b$, with the corresponding phase factor ρ and the tilde denoting 1 or 0.

In this way, more general results can be similarly obtained by starting from some more general many-BBO configurations, such as the conditional creation of single- or two-mode m-photo-added ECS. In addition, by replacing some BBO as the beam splitters (BS) in Fig. 1, one can also realize the photon-subtraction operations just as Parigi et al. showed in their recent BBO-BS-based experiment [11–13].

Now we proceed to discuss the impacts of input thermal noises on the PACS/PAECS generation by focusing on the case of only signal noises in the single-BBO case for simplicity. The finite temperature effect is described by the Takahashi-Umezawa formalism of thermo-field dynamics (TFD) in which the thermo vacuum is defined as [19]: $|0\rangle_T \equiv \hat{H}(\theta)|0\tilde{0}\rangle$, where the new vacuum $|0\tilde{0}\rangle$ belongs to the double Hilbert space determined by the tilde conjugate, and the heating operator: $\hat{H}(\theta) = \exp[-\theta(\hat{a}_s\hat{\tilde{a}}_s - \hat{a}_s^\dagger\hat{\tilde{a}}_s^\dagger)]$ provides a familiar Bogoliubov transformation

$$\hat{H}^\dagger(\theta)\hat{a}_s\hat{H}(\theta) \equiv \hat{b}_s = u(\beta)\hat{a}_s + v(\beta)\hat{\tilde{a}}_s^\dagger, \tag{7}$$

where $u(\beta) = \cosh\theta$, $v(\beta) = \sinh\theta$ and the new quasi-particle operators satisfy the standard bosonic commutation relation: $[\hat{b}_s, \hat{b}_s^\dagger] = 1$. The thermal vacuum photons obey the Bose-Einstein distribution

$$\bar{n} \equiv \langle 0\tilde{0}|\hat{b}_s^\dagger\hat{b}_s|0\tilde{0}\rangle = \sinh^2\theta = (e^{\beta\omega} - 1)^{-1}, \tag{8}$$

with $\beta = (k_B T)^{-1}$, $u(\beta) = \sqrt{\bar{n}+1}$ and $v(\beta) = \sqrt{\bar{n}}$. This expression determines the heating coefficient θ in the heating operator. Now we can take the initial low-temperature ($v(\beta) \ll u(\beta)$) input state as a mixed coherent-thermal field [19]: $|in\rangle_s = \hat{H}(\theta)\hat{D}(\alpha)|0\tilde{0}\rangle_s = \hat{H}(\theta)|\alpha\tilde{0}\rangle_s$ (an additional fictitious displacement operator can also be introduced for a higher temperature). Then the final output state can be obtained as (up to the first order of λ)

$$|\psi(t)\rangle_1 = \hat{H}(\theta)\exp[\lambda(\hat{b}_s^\dagger\hat{a}_i^\dagger - \hat{b}_s\hat{a}_i)]|\alpha\tilde{0}\rangle_s|0\rangle_i \approx |in\rangle_s|0\rangle_i + \lambda u(\beta)\hat{H}(\theta)|\alpha\tilde{0}, 1\rangle_s|1\rangle_i. \tag{9}$$

This, by ignoring the unchanged mode, leads to a conversion: $\hat{H}(\theta)|\alpha\rangle_s|0\rangle_i \rightarrow \hat{H}(\theta)|\alpha, 1\rangle_s|1\rangle_i$ with the success probability of $p_1^1(\beta) = \lambda u(\beta)$, which means that, even with the input mixed coherent-thermal fields, the SPACS still can be achieved with an enhanced success probability: $\lambda \rightarrow \lambda u(\beta)$. The effect of the thermal noise is the achieved thermalized SPACS instead of an ideal SPACS. Similar results can be readily obtained for creating, e.g., the EPACS under the repeated-BBO configuration.

Finally we give a comparison of the entanglement degree between the single- or two-mode photon-added ECS, the bare ECS, and even the photon-added-and-subtracted ECS. The later state can be viewed as a generalized photon-added-and-subtracted coherent state of Parigi et al. [11–13]. To this end, we introduce the concurrence of the bipartite entangled state $|\psi\rangle = \frac{1}{N}(\mu|\phi_1\rangle|\sigma_2\rangle + \nu|\varphi_1\rangle|\varsigma_2\rangle)$ as [20]

$$\mathcal{C}(|\psi\rangle) = \frac{2|\mu|\nu}{N^2}\sqrt{(1-|p_1|^2)(1-|p_2|^2)} \quad (p_1 = \langle\phi_1|\varphi_1\rangle,\ p_2 = \langle\varsigma_2|\sigma_2\rangle), \tag{10}$$

with the normalized constant $N^2 = |\mu|^2 + |\nu|^2 + 2Re(\mu^*\nu p_1 p_2^*)$. Figure 2 shows an example ($\beta = 0$) of the computed concurrence of single-mode or two-mode photon-added or photon-added-and-subtracted ECS and the bare ECS. Clearly, we have the following interesting results:

$$\begin{aligned}
&\mathcal{C}(\hat{a}^\dagger\hat{a}|ECS\rangle) > \mathcal{C}(|ECS\rangle) > \mathcal{C}(\hat{a}\hat{a}^\dagger|ECS\rangle),\\
&\mathcal{C}(TMECS\rangle) > \mathcal{C}(|SMECS\rangle),\\
&\mathcal{C}(\hat{a}\hat{b}\hat{a}^\dagger\hat{b}^\dagger|ECS\rangle) > \mathcal{C}(\hat{a}^\dagger\hat{b}^\dagger|ECS\rangle) > \mathcal{C}(\hat{a}\hat{a}^\dagger|ECS\rangle),
\end{aligned} \tag{11}$$

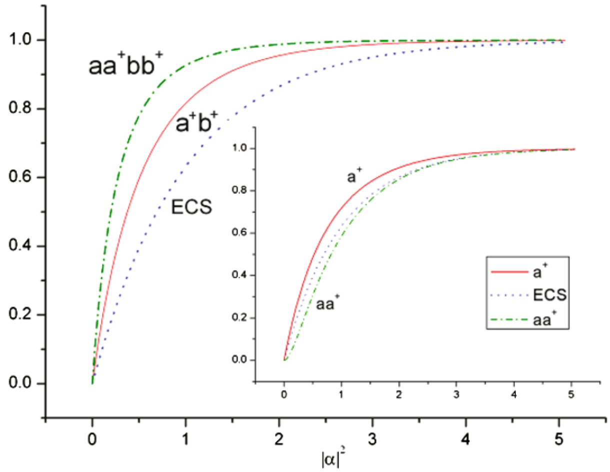

Fig. 2 (Color online) The concurrence of the two-mode two-photon-added ECS, the bare ECS, and the two-mode photon-added-and-subtracted ECS [11–13]. The *inset* is the single-mode single-photon-added ECS and the single-mode photon-added-and-subtracted ECS. Note that the lines of the single- or two-mode photo-added-and-subtracted ECS are below or above the TMECS/ECS lines, respectively

which means that both of the results of single-mode and two-mode photon-added ECS are different from that of the bare ECS, and more interestingly, the results of the single- or two-mode photo-added-and-subtracted ECS can be completely different (below or above the TMECS/ECS lines).

4 Conclusion

In conclusion, we propose a feasible scheme to create the single- or two-mode photon-added entangled coherent state. The different entanglement degrees of various states, such as the single-mode SPACS, the two-mode two-photon photon-added ECS, the bare ECS and even the single- or two-mode photon-added-and-subtracted ECS, are compared. The apparently differences of these novel quantum states indicate a potentially useful method to create and manipulate the non-classicality of the ECS or the photo-added ECS. Of course, there are some realistic problems such as the imperfect elements which affects the generation efficiency and the fidelity of the desired output state, and many authors analyzed in detail such losses as well as the suggestions of improving the efficiency and fidelity [21–24]. Future works will study these issues, the optimal scheme to create the photon-added-and-subtracted ECS, and their possible applications in current quantum information science.

Acknowledgement This work was supported partially by NSFC and Henan Talented Youth Program.

References

1. Scully, M.O., Zubairy, M.S.: Quantum Optics. Cambridge University Press, Cambridge (1997)
2. Said, R.S., Wahiddin, M.R.B., Umarov, B.A.: J. Phys. B At. Mol. Opt. Phys. **39**, 1269 (2006)
3. Sanaka, K., Resch, K.J., Zeilinger, A.: Phys. Rev. Lett. **96**, 083601 (2006)
4. Lvovsky, A.I., Babichev, S.A.: Phys. Rev. A **66**, 011801(R) (2002)
5. Escher, B.M., Avelar, A.T., Baseia, B.: Phys. Rev. A **72**, 045803 (2005)
6. Li, Y., Jing, H., Zhan, M.: J. Phys. B At. Mol. Opt. Phys. **39**, 2107 (2006)
7. Sanders, B.C.: Phys. Rev. A **45**, 6811 (1992)
8. Agarwal, G.S., Tara, K.: Phys. Rev. A **43**, 492 (1991)
9. Zavatta, A., Viciani, S., Bellini, M.: Science **306**, 660 (2004)

10. Parigi, V., Zavatta, A., Kim, M., Bellini, M.: Science **317**, 1890 (2007)
11. Scarani, V., Gisin, N.: Phys. Rev. Lett. **87**, 117901 (2001)
12. Eibl, M., et al.: Phys. Rev. Lett. **90**, 200403 (2003)
13. Zhao, Z., et al.: Phys. Rev. Lett. **91**, 180401 (2003)
14. Wolf, E., Mandel, L.: Optical Coherence and Quantum Optics. Cambridge University Press, Cambridge (1995)
15. Bose, S., Home, D.: Phys. Rev. Lett. **88**, 050401 (2002)
16. Campos, R.A.: Phys. Rev. A **62**, 013809 (2000)
17. Hau, L.V., et al.: Nature (London) **397**, 594 (1999)
18. Lukin, M.D.: Rev. Mod. Phys. **75**, 457 (2003)
19. Barnett, S.M., Knight, P.L.: J. Opt. Soc. Am. B **2**, 467 (1985)
20. Kuang, L.-M., Zhou, L.: Phys. Rev. A **68**, 043606 (2003)
21. Paris, M.G.A.: Phys. Rev. A **62**, 033813 (2000)
22. Pegg, D.T., Phillips, L.S., Barnett, S.M.: Phys. Rev. Lett. **81**, 1604 (1998)
23. Zou, X.B., Pahlke, K., Mathis, W.: Phys. Rev. A **66**, 014102 (2002)
24. Gardiner, C.W., Zoller, P.: In Haken, H. (ed.) Quantum Noise. A Handbook of Markovian and Non-Markovian Quantum Stochastic Methods with Applications to Quantum Optics, p. 397 (2000)